You maintained two free checking accounts for several years. **SCORE: _____ / 2 PTS**

[a] On the 26^{th} day that the first account was opened, you started direct deposit of your various paychecks into it. On the 1409^{th} day that that account was opened, you stopped the direct deposit. For how many days were you using that account for direct deposit ?

1 $\underline{1409-26+1=1384}$

[b] You opened the second account with $\$900$, but after an argument with customer service one day, you withdrew almost all the funds that day, leaving only $1¢$ in the account. On the 1278^{th} day that that account was opened, you closed the account. If your balance had been $1¢$ for 653 days, on which day did you have the argument with customer service ?

1 $\underline{1278-x+1=653 \qquad x=626}$

Remember that a code may begin with leading 0's, but an integer must not. **SCORE: _____ / 8 PTS**

[a] How many 6 digit positive **codes** contain 3 digits which are the same as each other, and 3 other digits which are the same as each other but different from the other 3 digits ?

1 Choose 2 digits to be used	$\frac{1}{2}$ $C(10, 2)$ ways
1 Choose 3 positions for the smaller digit	$\frac{1}{2}$ $C(6, 3)$ ways
Put the larger digit into the other 3 positions	1 way
	Total $= C(10, 2) \cdot C(6, 3) = 900$ codes $\frac{1}{2}$

[b] How many 6 digit positive **integers** contain 3 digits which are the same as each other, and 3 other digits which are the same as each other but different from the other 3 digits ?

1 Choose 1 digit (not 0) for leading digit	$\frac{1}{2}$ 9 ways
1 Choose 2 other positions for that digit	$\frac{1}{2}$ $C(5, 2)$ ways
$\frac{1}{2}$ Choose a different digit for the other positions	$\frac{1}{2}$ 9 ways
	Total $= 9 \cdot C(5, 2) \cdot 9 = 810$ integers $\frac{1}{2}$

Consider all the functions with domain $A = \{a_1, a_2, a_3, \ldots, a_k\}$ and co-domain $B = \{b_1, b_2, b_3, \ldots, b_n\}$.

SCORE: _____ / 6 PTS

[a] How many such functions are there ?

1

Choose an element of B to be the image of a_1 — n ways
Choose an element of B to be the image of a_2 — n ways
Choose an element of B to be the image of a_3 — n ways
$\vdots$ — $\vdots$
Choose an element of B to be the image of a_k — n ways

1

Total $= \boxed{n^k}$ functions

1

[b] If $k \le n$, how many such functions are one-to-one ?

1

Choose an element of B to be the image of a_1 — n ways
Choose an element of B to be the image of a_2 — $n-1$ ways
(but do not choose the previously chosen image of a_1)
Choose an element of B to be the image of a_3 — $n-2$ ways
(but do not choose the previously chosen images of a_1, a_2)
$\vdots$ — $\vdots$
Choose an element of B to be the image of a_k — $n-k+1$ ways
(but do not choose the previously chosen images of $a_1, a_2, a_3, \ldots, a_{k-1}$)

1

Total $= n(n-1)(n-2)\ldots(n-k+1) = P(n, k)$ functions

1

ALTERNATE SOLUTION:
Arrange k elements of B in a line — $P(n, k)$ ways
(the element in position i will be image of a_i for $i = 1, 2, 3, \ldots, k$)

Nine cards are selected from a standard deck of cards to form a hand. **SCORE: _____ / 19 PTS**

[a] How many hands contain only diamonds ?

$C(13, 9)$ 1

[b] How many hands contain no diamonds ?

$C(52-13, 9) = C(39, 9)$ 1

[c] How many hands contain cards from at least 2 different suits ?

½ Choose a suit	½ 4 ways
½ Choose 9 cards from that suit	½ $C(13, 9)$ ways

TOTAL $= 4 \cdot C(13, 9)$ hands that contain cards from only 1 suit ½

$C(52, 9)$ – number of hands that contain cards from only 1 suit $= C(52, 9) - 4 \cdot C(13, 9)$ hands ½

[d] How many hands contain at least 1 diamond and 1 heart (simultaneously) ?

Let $A = \{$hands with no diamonds$\}$ — $|A| = C(39, 9)$ from [b] ½

Let $B = \{$hands with no hearts$\}$ — $|B| = C(39, 9)$ from same logic in [b] ½

$A \cap B = \{$hands with no diamonds and no hearts simultaneously$\}$ — $|A \cap B| = C(52-26, 9) = C(26, 9)$ 1

$A \cup B = \{$hands with no diamonds or no hearts$\}$ — $|A \cup B| = |A| + |B| - |A \cap B|$

$= 2 \cdot C(39, 9) - C(26, 9)$ 2

$(A \cup B)^C = \{$hands with at least 1 diamond and 1 heart simultaneously$\}$ — $= C(52, 9) - (2 \cdot C(39, 9) - C(26, 9))$ ½

IF YOU GOT THIS ANSWER, BUT DID NOT BREAK IT DOWN INTO ALL THE STEPS, TAKE ALL 4½ POINTS LISTED ABOVE

[e] How many hands contain 6 cards from the same suit, and 3 cards from another suit ?
(eg. 2♥, 4♥, 5♥, 9♥, J♥, Q♥, A♦, 5♦, J♦)

½ Choose a suit for the 6-of-a-suit	½ 4 ways
½ Choose 6 cards of that suit	½ $C(13, 6)$ ways
½ Choose a different suit for the 3-of-a-suit	½ 3 ways
½ Choose 3 cards of that suit	½ $C(13, 3)$ ways

Total $= 4 \cdot C(13, 6) \cdot 3 \cdot C(13, 3)$ hands ½

[f] How many hands contain 3 cards from each of 3 suits ?
(eg. 2♥, 4♥, 5♥, 4♣, 9♣, Q♣, A♦, 5♦, J♦)

1 Choose 3 suits	½ $C(4, 3)$ ways
½ Choose 3 cards for the (alphabetically) first suit	½ $C(13, 3)$ ways
½ Choose 3 cards for the (alphabetically) second suit	½ $C(13, 3)$ ways
½ Choose 3 cards for the (alphabetically) third suit	½ $C(13, 3)$ ways

Total $= C(4, 3) \cdot C(13, 3) \cdot C(13, 3) \cdot C(13, 3)$ hands ½