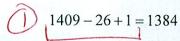
[a] On the 26<sup>th</sup> day that the first account was opened, you started direct deposit of your various paychecks into it. On the 1409<sup>th</sup> day that that account was opened, you stopped the direct deposit. For how many days were you using that account for direct deposit?



You opened the second account with \$900, but after an argument with customer service one day, you withdrew almost all the funds that day, leaving only  $1 \, \text{¢}$  in the account. On the  $1278^{th}$  day that that account was opened, you closed the account. If your balance had been  $1 \, \text{¢}$  for 653 days, on which day did you have the argument with customer service?

$$1278 - x + 1 = 653 \qquad x = 626$$

- [a] How many 6 digit positive <u>codes</u> contain 3 digits which are the same as each other, and 3 other digits which are the same as each other but different from the other 3 digits?
  - Choose 2 digits to be used
    - Choose 3 positions for the smaller digit

      Put the larger digit into the other 3 positions

C(10, 2) ways C(6, 3) ways C(6, 3) ways

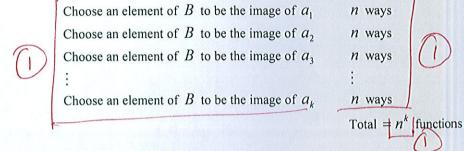
Total  $= C(10, 2) \cdot C(6, 3) = 900$  codes

- [b] How many 6 digit positive <u>integers</u> contain 3 digits which are the same as each other, and 3 other digits which are the same as each other but different from the other 3 digits?
  - Choose 2 other positions for that digit Choose a different digit for the other positions

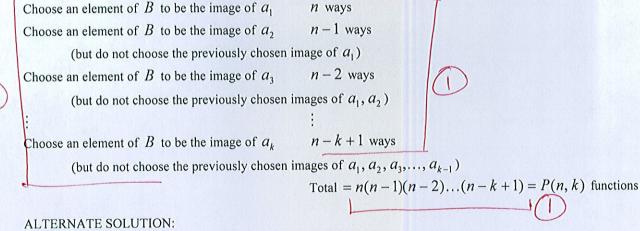
C(5, 2) ways

Total =  $9 \cdot C(5, 2) \cdot 9 = 810$  integers

How many such functions are there? [a]



If  $k \le n$ , how many such functions are one-to-one? [b]



P(n,k) ways Arrange k elements of B in a line (the element in position i will be image of  $a_i$  for i = 1, 2, 3, ..., k)

Vine cards are selected	from a standard	deck of cards to form a hand.	
ville calus are selected	from a standard	deck of cards to form a hand.	

SCORE: / 19 PTS

[a] How many hands contain only diamonds?

C(13, 9)

[b] How many hands contain no diamonds?

C(52-13,9) = C(39,9)

[c] How many hands contain cards from at least 2 different suits?

> Choose a suit Choose 9 cards from that suit

C(13, 9) ways

TOTAL  $4 \cdot C(13, 9)$  hands that contain cards from only 1 suit

C(52, 9) – number of hands that contain cards from only 1 suit =  $C(52, 9) - 4 \cdot C(13, 9)$  hands

[d] How many hands contain at least 1 diamond and 1 heart (simultaneously)?

Let  $A = \{ \text{ hands with no diamonds } \}$ 

Let  $B = \{ \text{ hands with no hearts } \}$ 

 $A \cap B = \{$  hands with no diamonds and no hearts simultaneously  $\}$ 

 $A \cup B = \{ \text{ hands with no diamonds or no hearts } \}$ 

 $(A \cup B)^C = \{ \text{ hands with at least 1 diamond and 1 heart simultaneously } \}$ 

|A| = C(39, 9) from [b] |B| = C(39, 9) from same logic in [b]

 $|A \cap B| = C(52 - 26, 9) = C(26, 9)$ 

 $|A \cup B| = |A| + |B| - |A \cap B|$ 

 $= 2 \cdot C(39, 9) - C(26, 9)$ 

 $= C(52, 9) - (2 \cdot C(39, 9) - C(26, 9))$ 

IF YOU GOT THIS ANSWER, BUT DID NOT BREAK IT

DOWN INTO ALL THE STEPS,

TAKE ALL 42 POINTS LISTED ABOVE

[e] How many hands contain 6 cards from the same suit, and 3 cards from another suit? (eg.  $2 \lor$ ,  $4 \lor$ ,  $5 \lor$ ,  $9 \lor$ ,  $J \lor$ ,  $Q \lor$ ,  $A \diamondsuit$ ,  $5 \diamondsuit$ ,  $J \diamondsuit$ )

Choose a suit for the 6-of-a-suit

Choose 6 cards of that suit

Choose a different suit for the 3-of-a-suit

Choose 3 cards of that suit

3 ways C(13, 3) ways

Total =  $4 \cdot C(13, 6) \cdot 3 \cdot C(13, 3)$  hands

[f]How many hands contain 3 cards from each of 3 suits? (eg.  $2 \checkmark$ ,  $4 \checkmark$ ,  $5 \checkmark$ ,  $4 \checkmark$ ,  $9 \checkmark$ ,  $Q \checkmark$ ,  $A \checkmark$ ,  $5 \checkmark$ ,  $J \checkmark$ )

Choose 3 suits

Choose 3 cards for the (alphabetically) first suit  $\int C(13, 3)$  ways

Choose 3 cards for the (alphabetically) second suit (C(13, 3)) ways Choose 3 cards for the (alphabetically) third suit C(13, 3) ways

C(4,3) ways

Total =  $C(4, 3) \cdot C(13, 3) \cdot C(13, 3) \cdot C(13, 3)$  hands